

# Frequency Planning By Means Of High-Dimensional Optimization

# Frequency Planning

- **frequency planning is not pure science**
- **basis is communication science and electrical engineering**
- **both technical and economical constraints, i.e. transmitter sites, erp and costs**
- **political constraints, different countries have different approaches to broadcasting**

# Frequency Plan Generation



## Allotment Plan

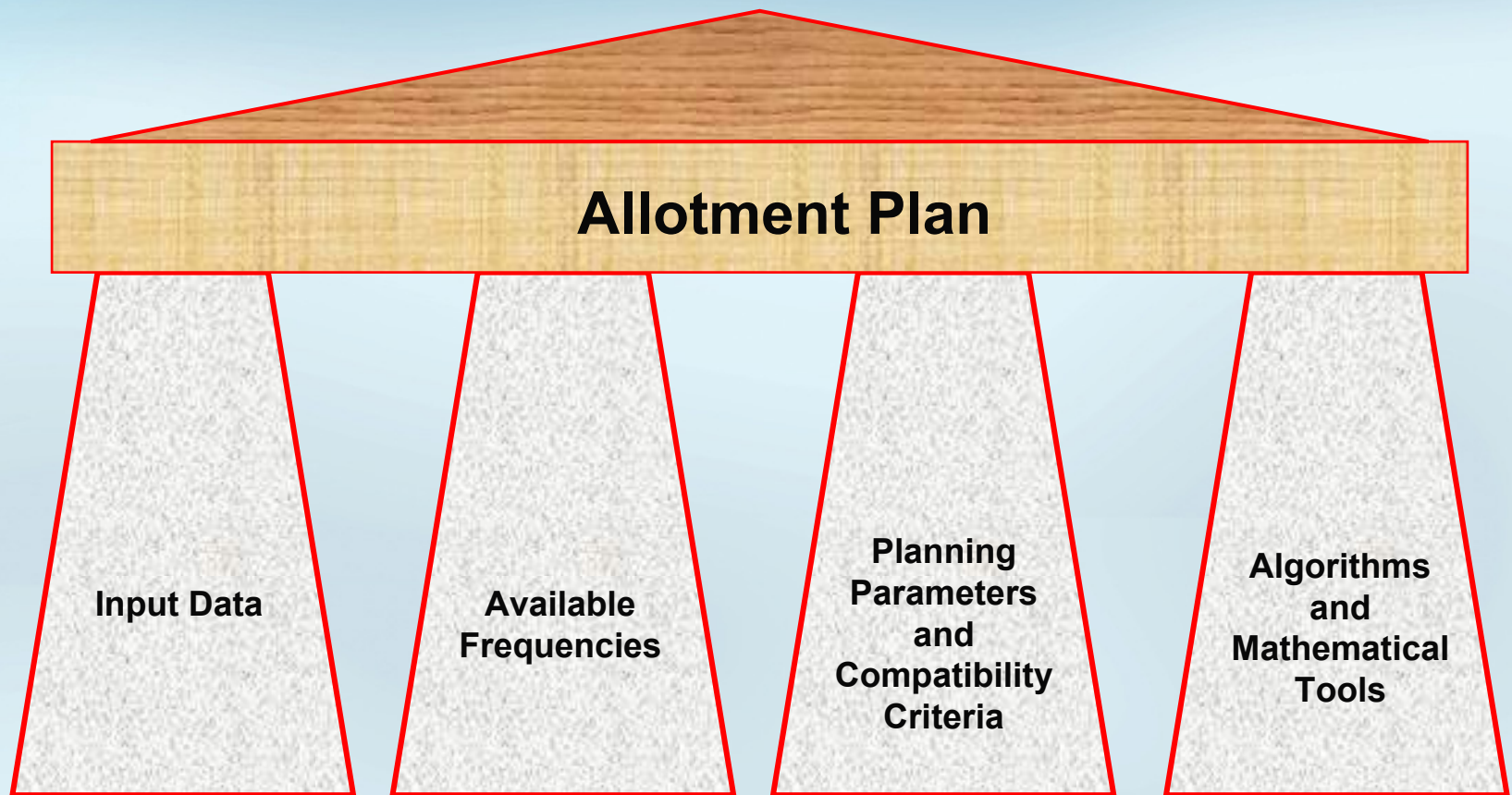
***Allotment (of a radio frequency or radio frequency channel):***

*Entry of a designated frequency channel in an agreed plan, adopted by a competent conference, for use by one or more administrations for a terrestrial or space radiocommunication service in one or more identified countries or geographical areas and under specified conditions.*

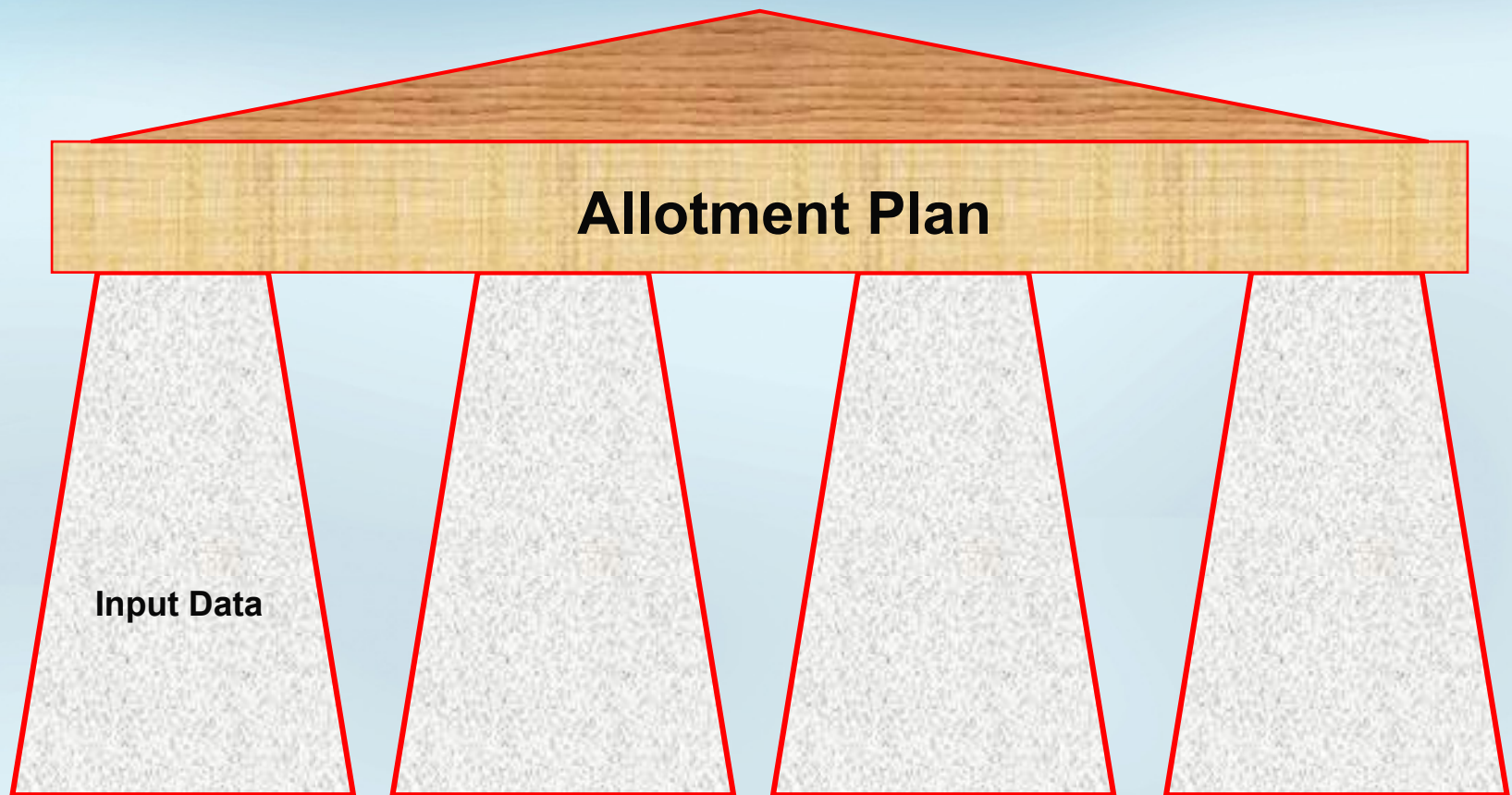
***Assignment (of a radio frequency or radio frequency channel):***

*Authorization given by an administration for a radio station to use a radio frequency or radio frequency channel under specified conditions.*

# Frequency Plan Generation



# Frequency Plan Generation



# Input Data

## Allotment Requirements:

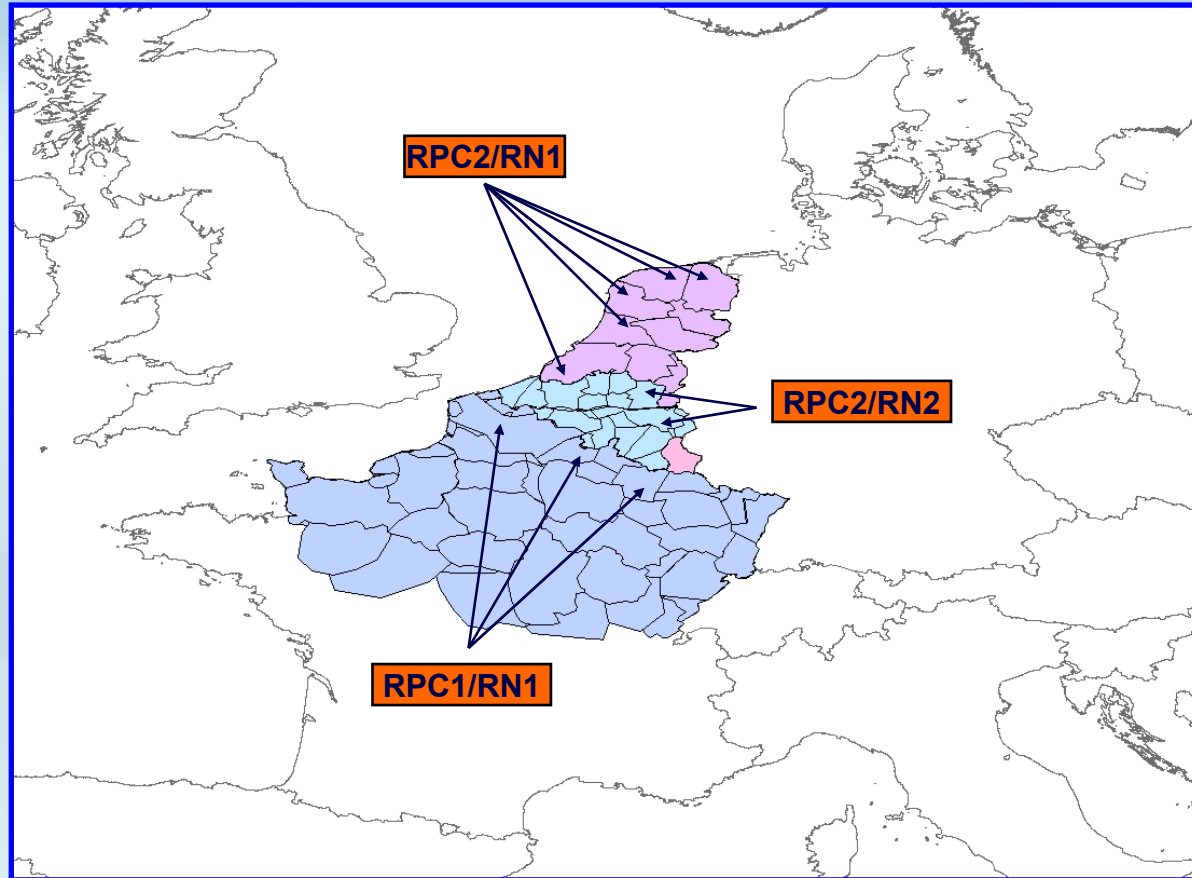
- polygons on a sphere, i.e. a finite number of vertices connected by segments of great circles

# Input Data

## Allotment Requirements:

- polygons on a sphere, i.e. a finite number of vertices connected by segments of great circles
- interference potential: reference networks (RN)  
coverage target : reference planning configurations (RPC)

# Input Data



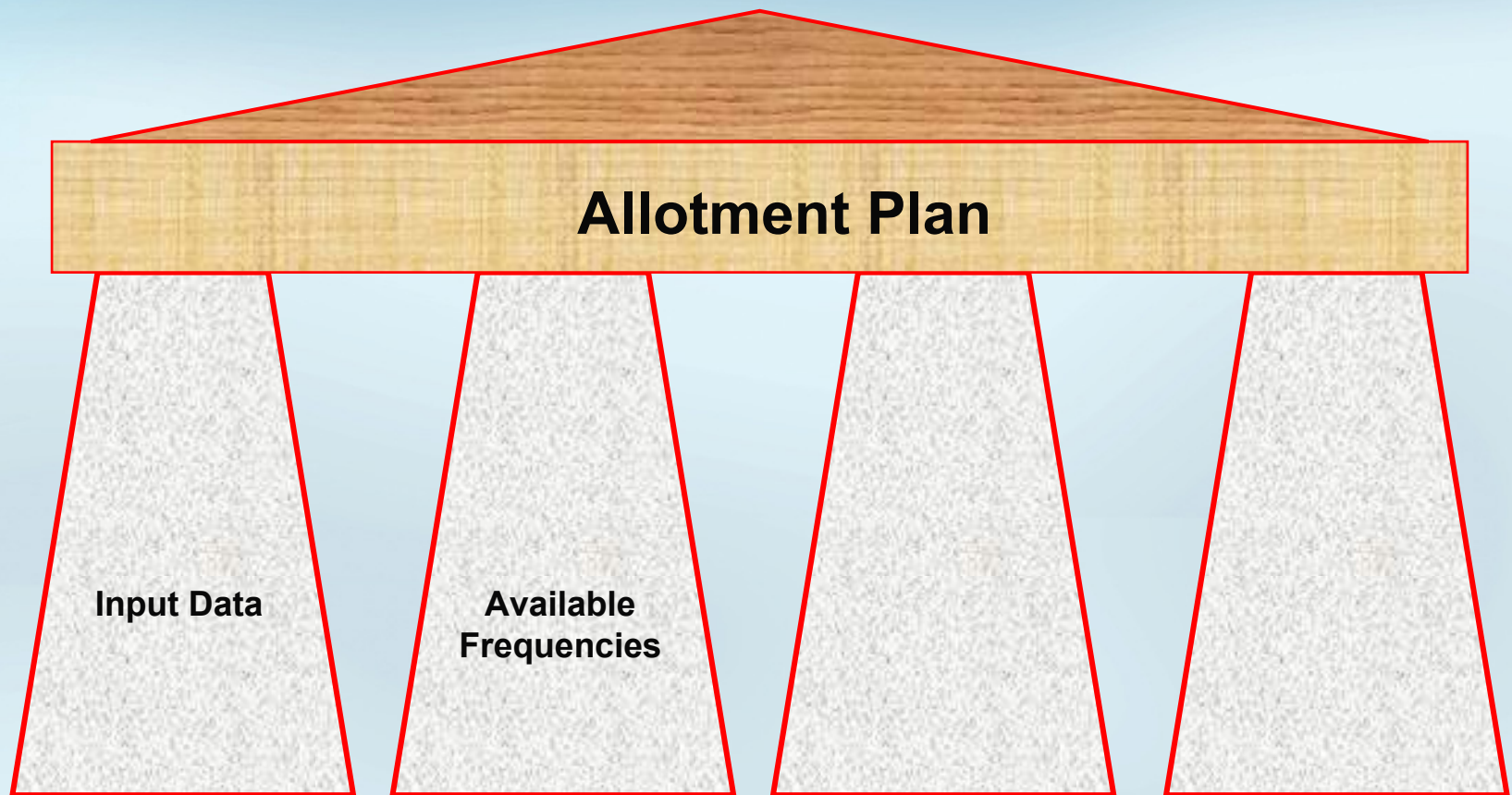


# Input Data

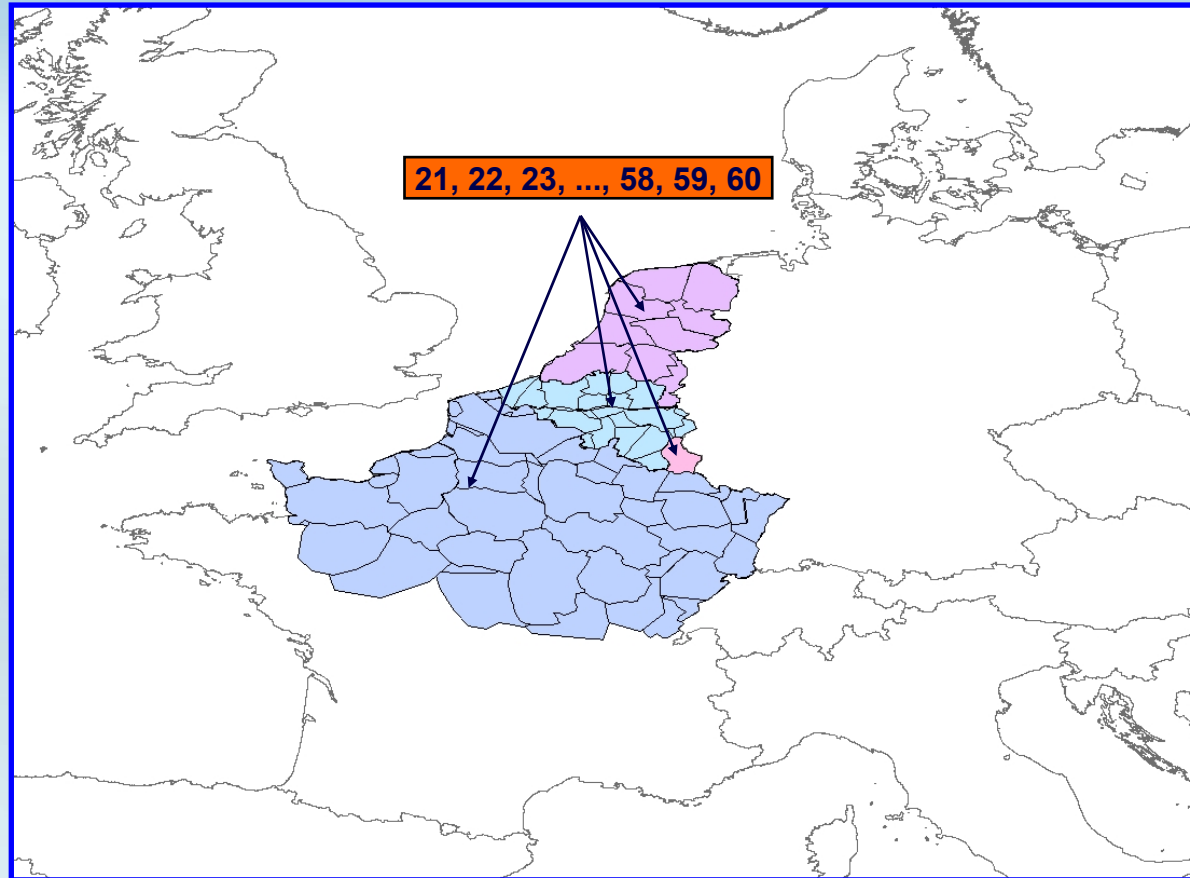
## Allotment Requirements:

- polygons on a sphere, i.e. a finite number of vertices connected by segments of great circles
- interference potential: reference networks (RN)  
coverage target : reference planning configurations (RPC)
- administrative declarations

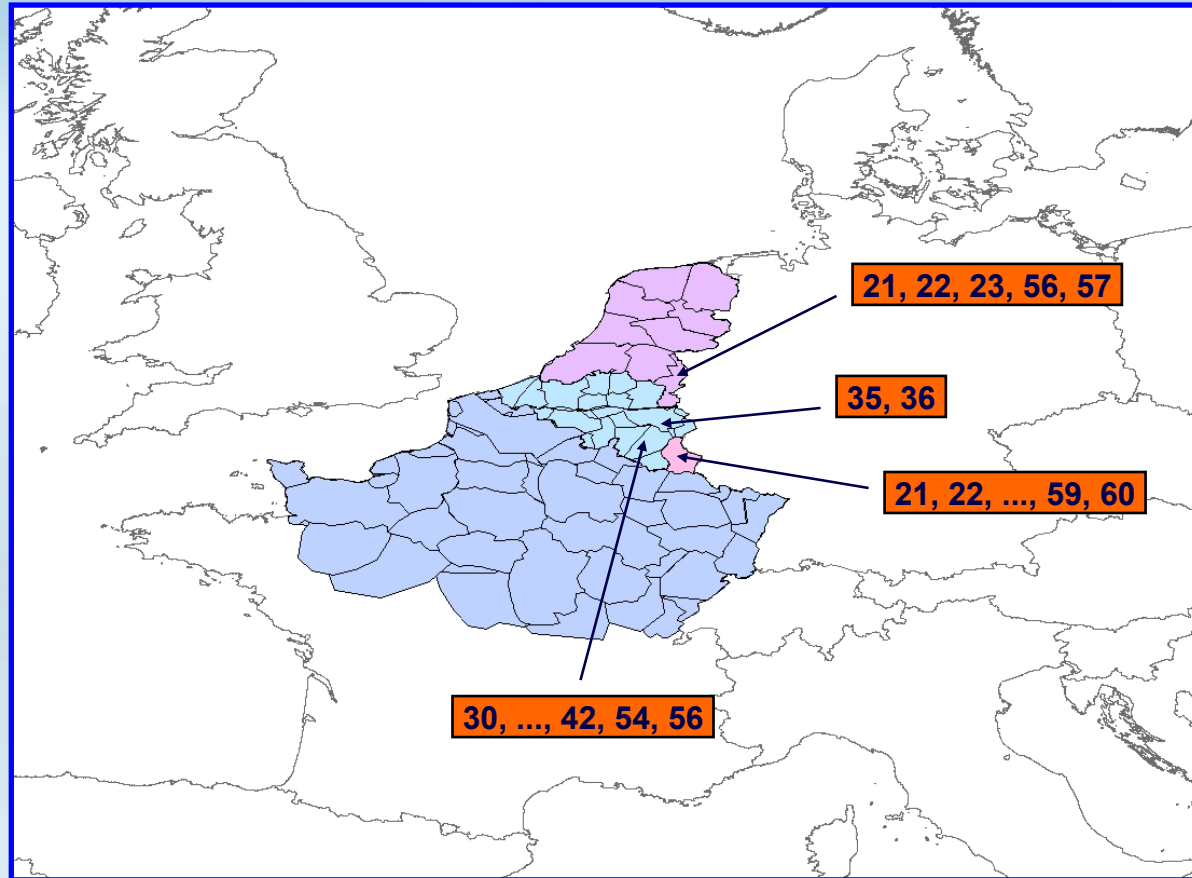
# Frequency Plan Generation



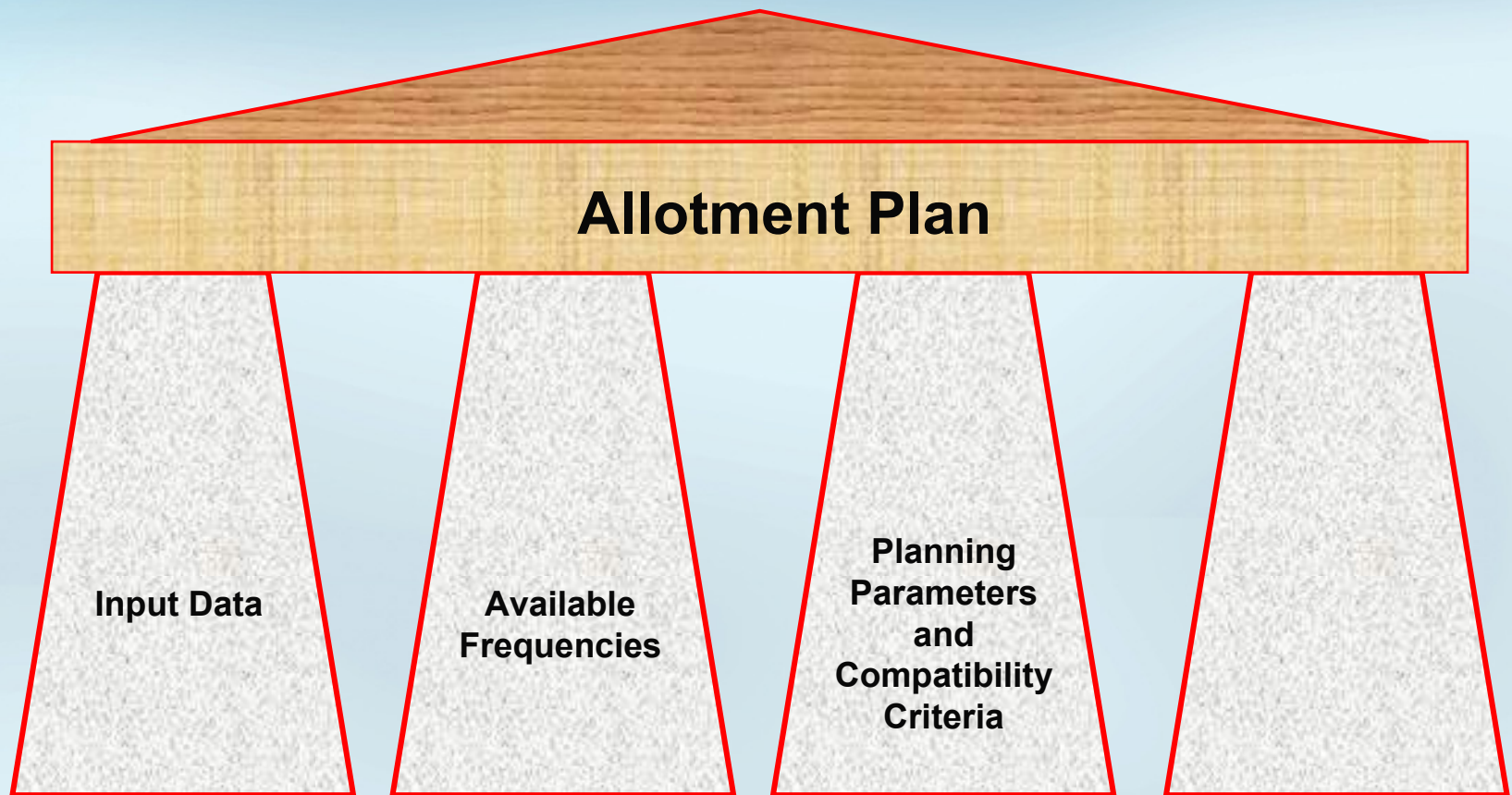
# Available Frequencies



# Available Frequencies



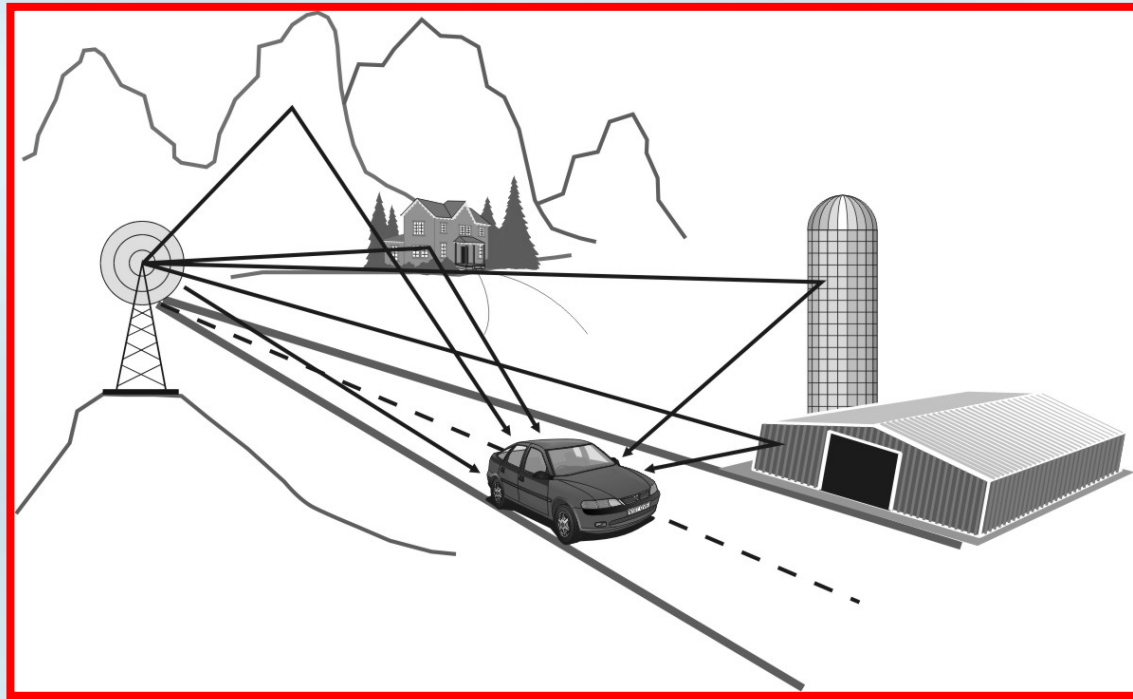
# Frequency Plan Generation



# Planning Parameters

## Wave Propagation

### Radio Channel

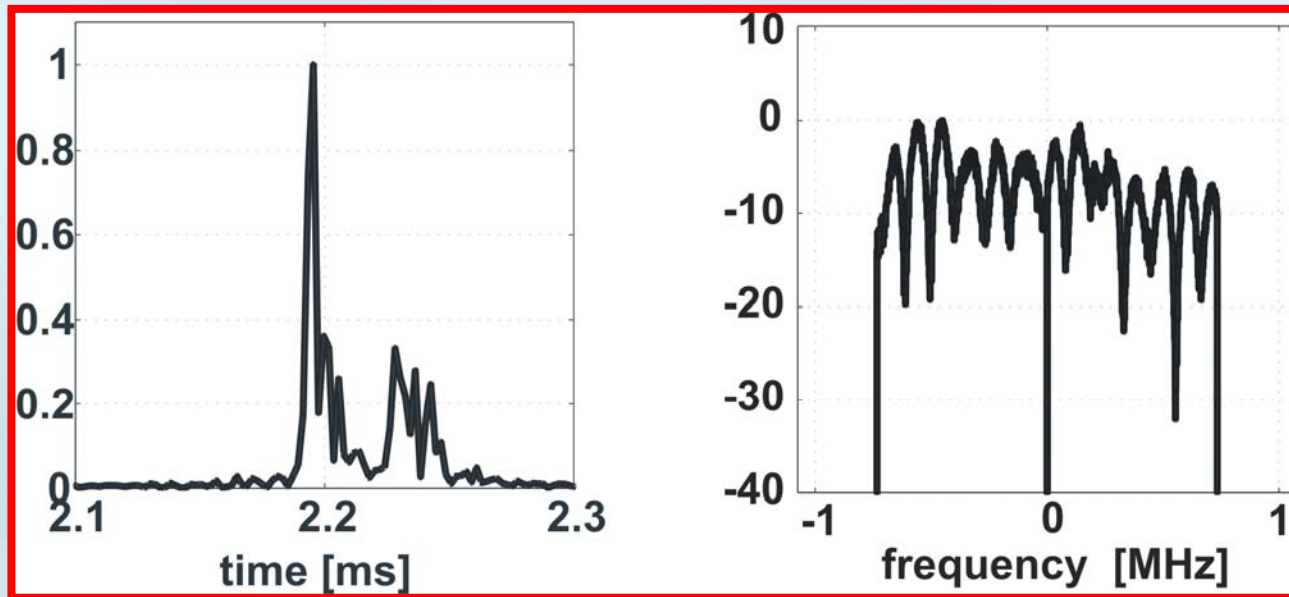


**multipath environment leading to frequency selective fading**

# Planning Parameters

## Wave Propagation

### Radio Channel



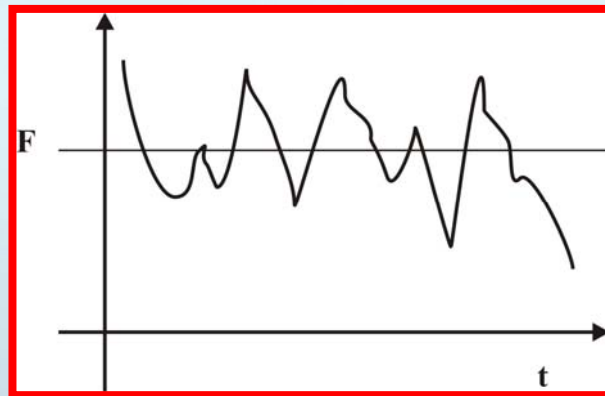
measurement of impulse response and transfer function of radio channel in DAB network near Stuttgart

# Planning Parameters

## Wave Propagation Model

**Task:** calculation of field strength at a given point

**Input data:** transmitter characteristics



→ temporal variation of radio channel requires statistical interpretation:

the value  $F$  corresponds to a field strength level which is exceeded in  $x$  % of time



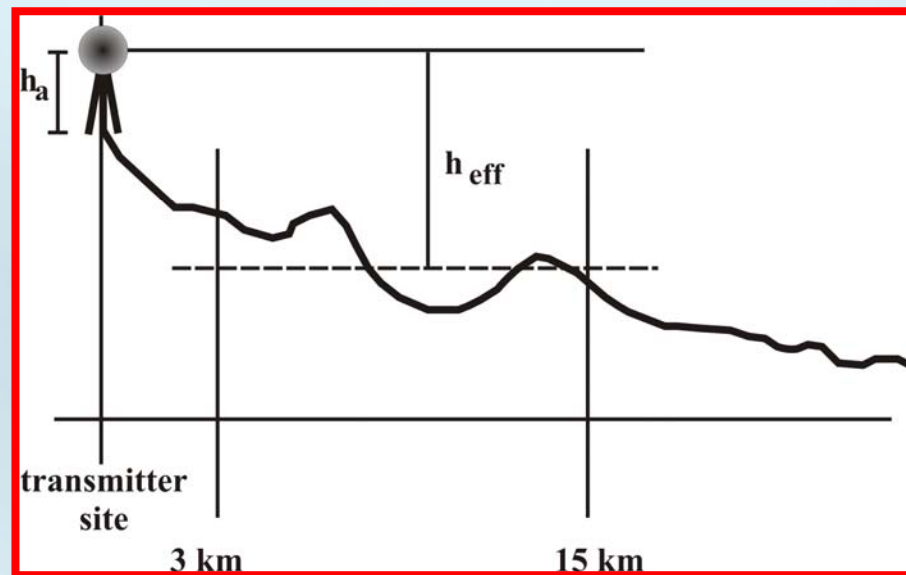
# Planning Parameters

## Wave Propagation Model

### ITU Recommendation P-1546

ITU Rec. P-1546: statistical model, not (really) path specific

Parameters: power, antenna height, antenna pattern, effective heights



# Planning Parameters

## Coverage Prediction

- **assessment of coverage quality in the presence of interference:**
  - required minimum field strength**
  - protection ratio**
- **problem: field strength calculation at discrete points only**
- **limited resolution of topographic and morphologic data defines „pixel“ size ( 100m\*100m )**
- **spatial distribution function (mean, standard deviation)**

# Planning Parameters

## Coverage Prediction

- calculation of probability that at x % of locations given field strength is exceeded

- coverage assessment in the presence of interference:

served: minimum field strength and protection ratio reached

interfered: minimum field strength reached, protection ratio not reached

not served: neither minimum field strength not protection ratio reached

} at x %  
of  
loc.

# Planning Parameters

## Reference Planning Configurations (RPC)

	<b>RPC1</b> (fixed reception)	<b>RPC2</b> (portable outdoor, mobile)	<b>RPC3</b> (portable indoor)
<b>system</b>	<b>DVB-T</b>	<b>DVB-T</b>	<b>DVB-T</b>
<b>location probability</b>	<b>95%</b>	<b>95%</b>	<b>95%</b>
<b>protection ratio</b>	<b>21 dB</b>	<b>19 dB</b>	<b>17 dB</b>
<b>minimum field strength at 200 MHz at 10 m</b>	<b>50 dB <math>\mu</math>V/m</b>	<b>67 dB <math>\mu</math>V/m</b>	<b>76 dB <math>\mu</math>V/m</b>
<b>minimum field strength at 650 MHz at 10 m</b>	<b>56 dB <math>\mu</math>V/m</b>	<b>78 dB <math>\mu</math>V/m</b>	<b>88 dB <math>\mu</math>V/m</b>

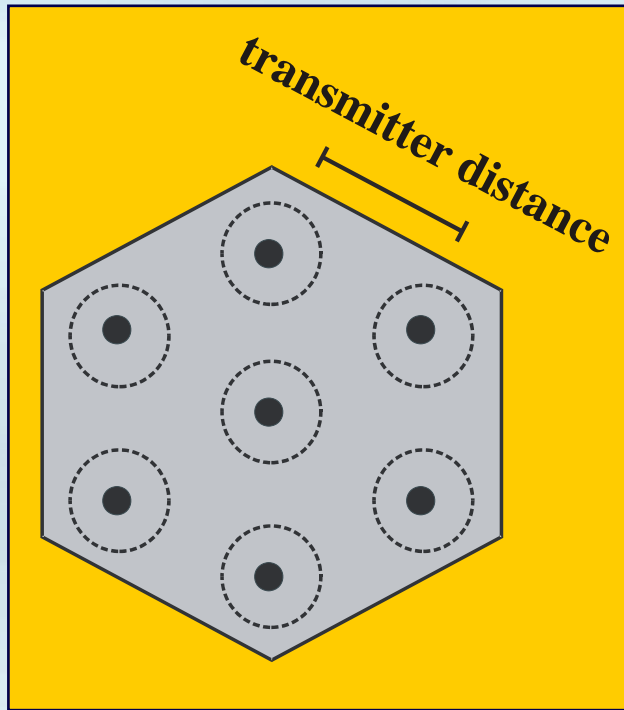
# Planning Parameters

## Reference Planning Configurations (RPC)

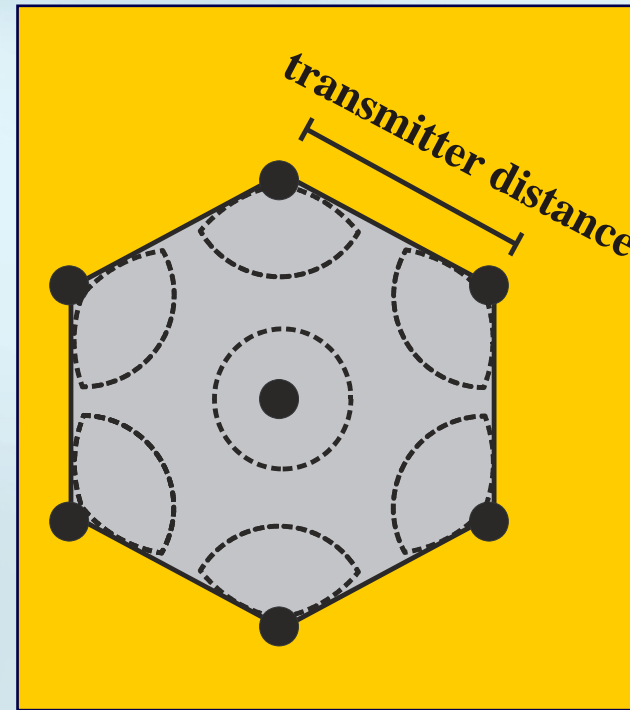
	<b>RPC4 (mobile)</b>	<b>RPC5 (portable indoor)</b>
<b>system</b>	<b>T-DAB</b>	<b>T-DAB</b>
<b>location probability</b>	<b>99%</b>	<b>95%</b>
<b>protection ratio</b>	<b>15 dB</b>	<b>15 dB</b>
<b>minimum field strength at 200 MHz at 10 m</b>	<b>60 dB V/m</b>	<b>66 dB V/m</b>

# Planning Parameters

## Reference Networks (RN)



open network



closed network

# Planning Parameters

## RN1 for DVB-T

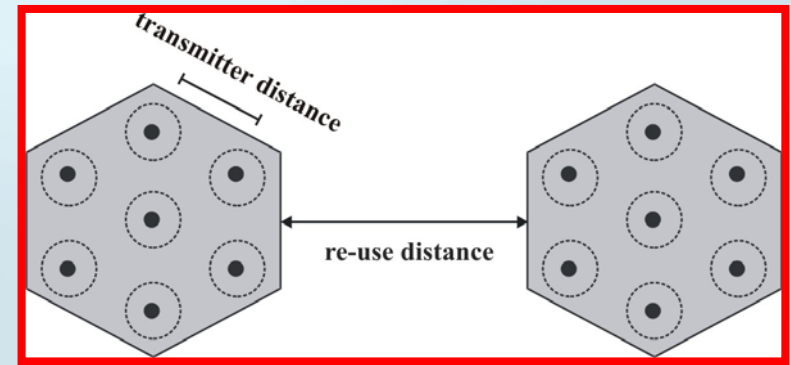
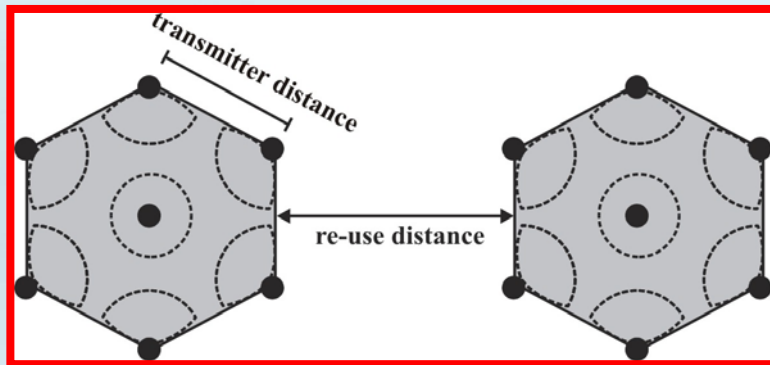
open hexagon, 7 transmitters

RPC and reception type	RPC 1 fixed antenna	RPC 2 portable outdoor and mobile	RPC 3 portable indoor
inter-transmitter distance	70 km	50 km	40 km
effective antenna height	150 m	150 m	150 m
ERP in Band III	4.1 dBkW (2.5 kW)	6.2 dBkW (4.2 kW)	10 dBkW (10 kW)
ERP in Band IV/V	12.8 dBkW (19 kW)	19.7 dBkW (93.3 kW)	22.4 dBkW (173.8 kW)

# Planning Parameters

## Re-Use Distances

**re-use distance:** minimum distance required between service areas using the same frequency providing different content





# Planning Parameters

## Re-Use Distances

**re-use distance:** minimum distance required between service areas using the same frequency providing different content

**calculation methods:**

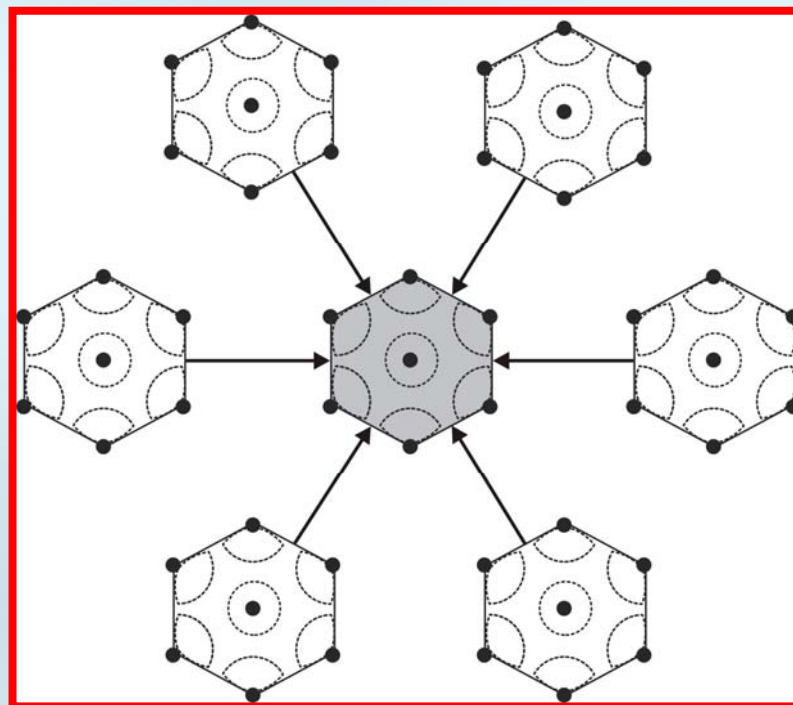
- a) „first pixel flips“
- b) infinite plane

# Planning Parameters

## Re-Use Distances

„first pixel flips“

- wanted network in the centre
- 6 interfering networks positioned on a hexagon
- distance changed from large to small
- re-use distance reached if first pixel inside wanted service area flips from „served“ to „interfered“



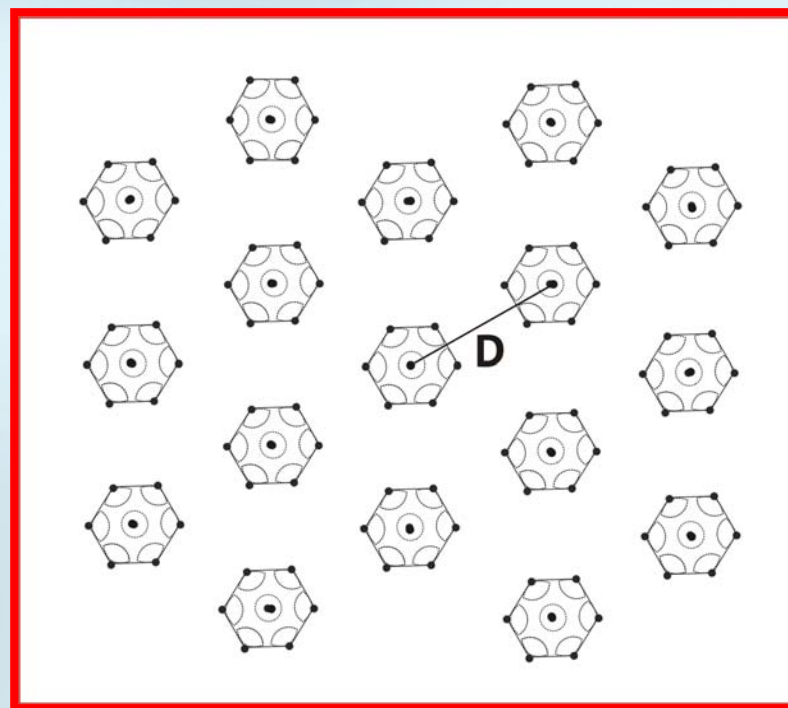
→ calculation of re-use distance protects coverage needs

# Planning Parameters

## Re-Use Distances

infinite plane

- „infinite number“ of networks on a hexagonal grid in the „infinite“ plane
- each pixel in large area is assessed and number of served pixels  $N$  is determined
- distance  $D$  is changed
- maximum of function  $D \rightarrow N$  defines re-use distance



→ calculation aims at optimal use of available spectrum

# Planning Parameters

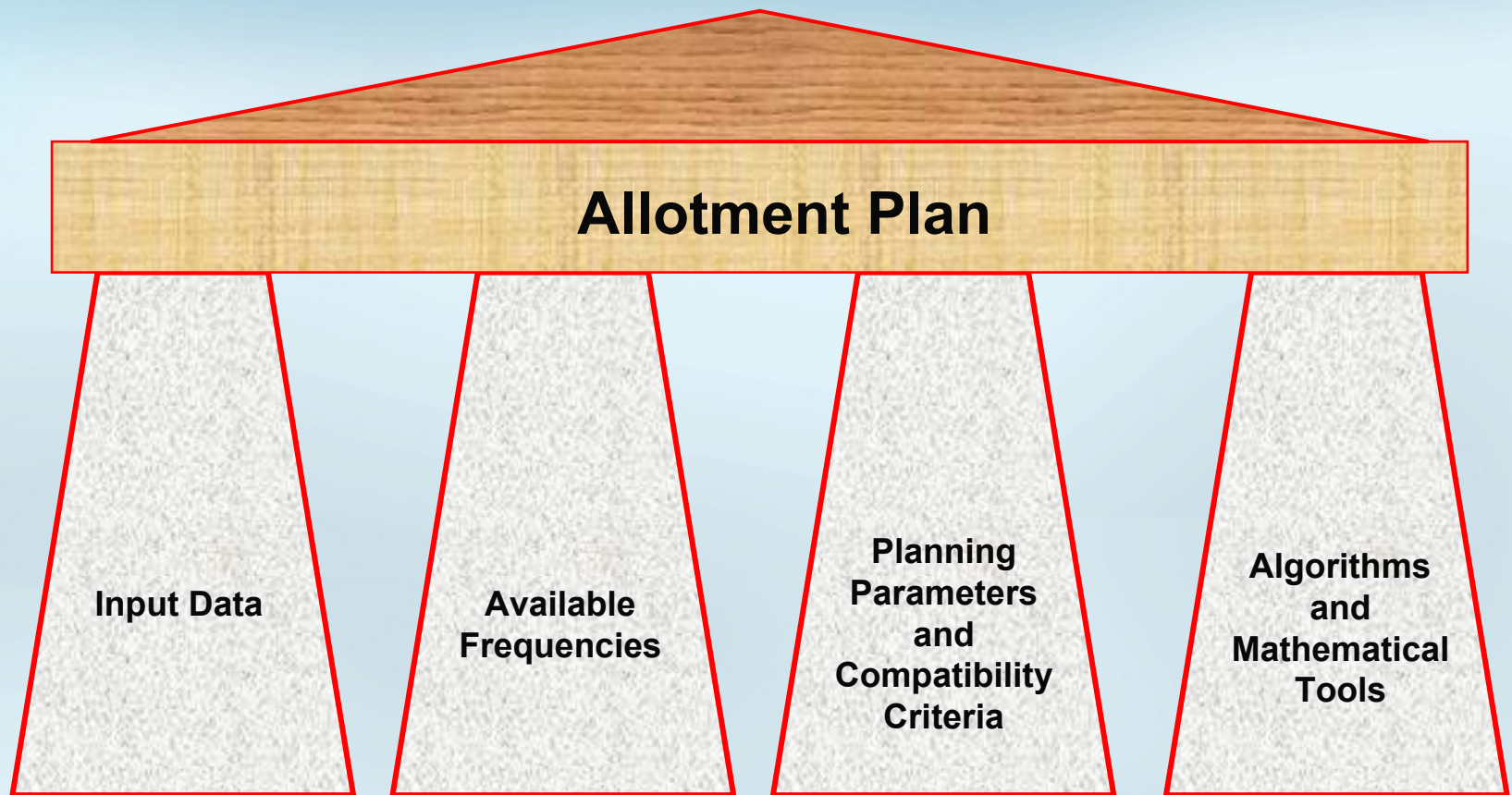
## Re-Use Distances propagation conditions

- ITU Rec. P-1546 covers propagation above land, cold sea and warm sea
- Re-use distance für T-DAB (WI95)

	land	cold sea	warm sea
VHF	81 km	142 km	173 km
L-Band	61 km	348 km	485 km

- Re-use distance für DVB-T                      100 km – 140 km

# Frequency Plan Generation



# Mathematical Algorithms

## General Approach of Frequency Planning Problems



iteration of process

# Mathematical Algorithms

## Compatibility Analysis

### Compatibility Analysis

assessment of pairs of requirements to determine if they can share a frequency



**Based on Field Strength**

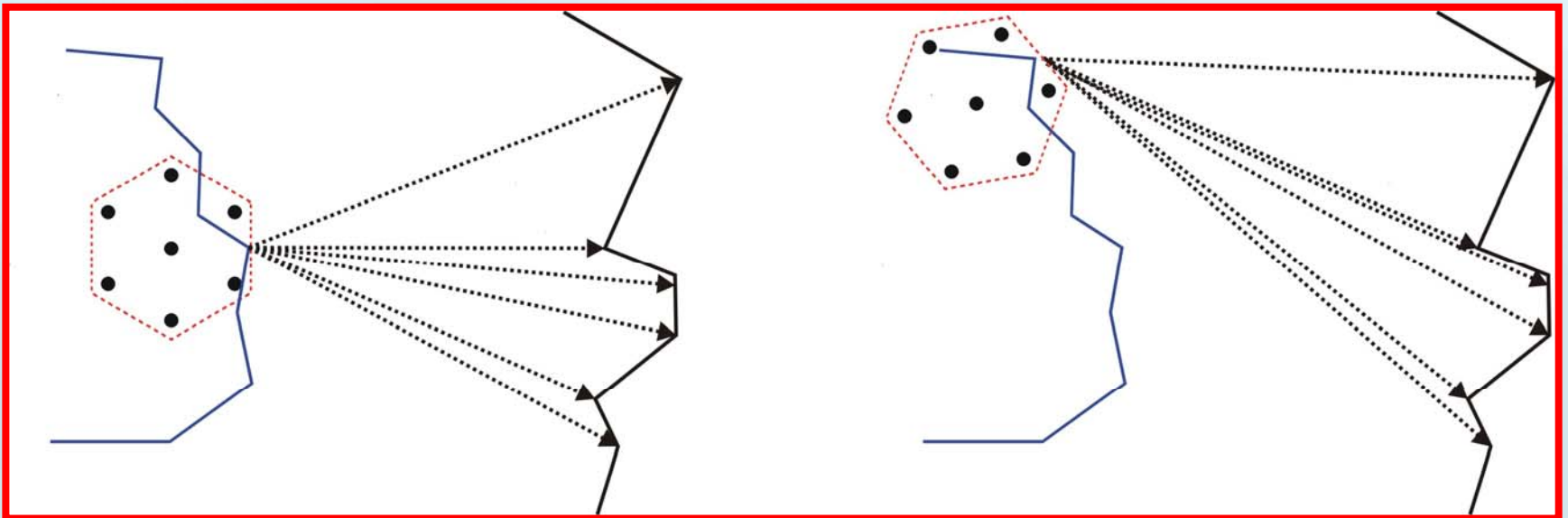


**Based on Effective Distance**

# Mathematical Algorithms

## Compatibility Analysis

### Field Strength Based



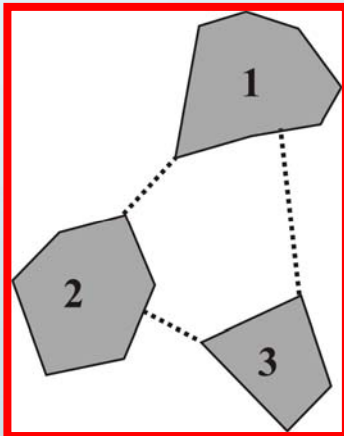


# Mathematical Algorithms

## Compatibility Analysis

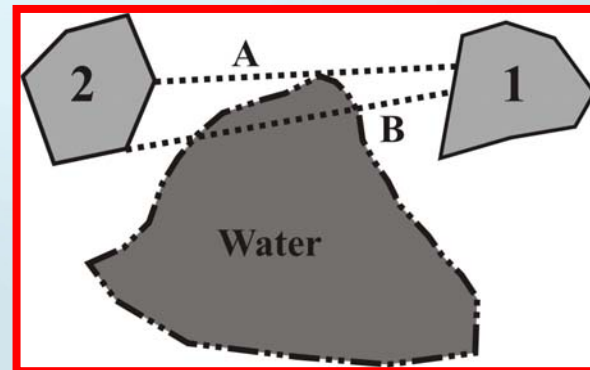
### Effective Distance

Idea: calculate distance between allotments and compare with re-use distance



minimal geometrical distance either between vertices or an edge and a vertex (also in spherical geometry!)

→ corresponds to most critical path



no longer true in the case of mixed propagation conditions !!!

# Mathematical Algorithms

## Compatibility Analysis

### Effective Distance

**Observation (VHF, T-DAB):** 81 km above land have the same impact as 142 km above cold water in VHF for T-DAB

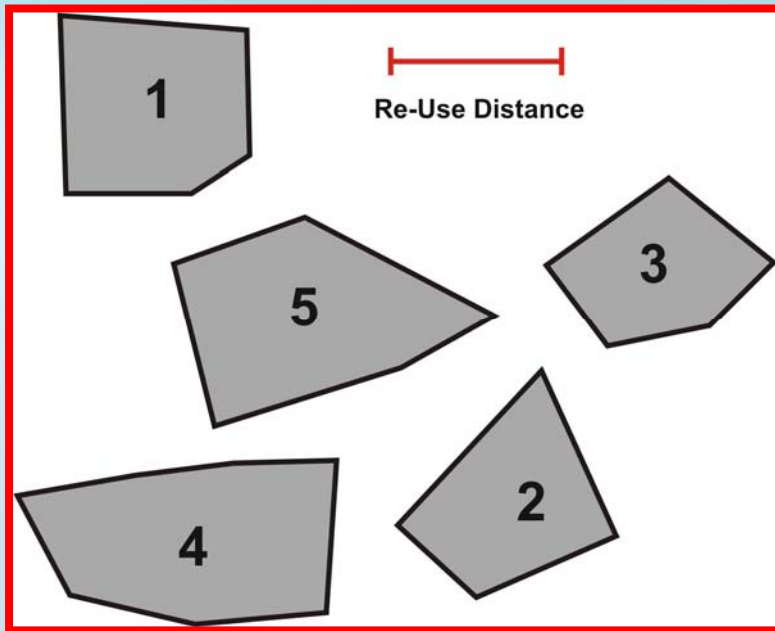
**Definition of Effective Distance between two points:**

- determine fractions of land (x), cold (y) and warm sea (z) path
- add different path lengths after appropriate scaling
- compare resulting effective distance with re-use distance above land

**Example (VHF, T-DAB):** 
$$d_{\text{eff}} = x + y * \frac{81}{142} + z * \frac{81}{173}$$

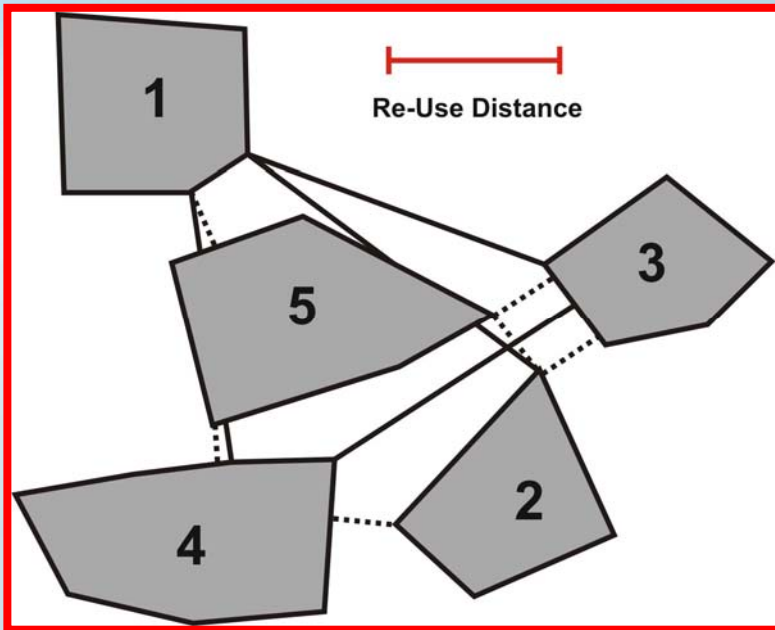
# Mathematical Algorithms

## Compatibility Analysis



# Mathematical Algorithms

## Compatibility Analysis



adjacency matrix

$$N = \begin{pmatrix} 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 1 & 1 \\ 0 & 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & 1 \\ 1 & 1 & 1 & 1 & 0 \end{pmatrix}$$



compatibility matrix

$$C = \begin{pmatrix} 0 & 48 & 41 & 35 & 9 \\ 48 & 0 & 9 & 8 & 10 \\ 41 & 9 & 0 & 37 & 8 \\ 35 & 8 & 37 & 0 & 5 \\ 9 & 10 & 8 & 5 & 0 \end{pmatrix}$$

# Mathematical Algorithms

## Compatibility Analysis

elements of adjacency matrix  $N$  :

- 0 or 1

elements of compatibility matrix  $C$  :

- effective distance
- ratio of area overlap of the two considered allotment areas



**$N$  and  $C$  are required for frequency assignment algorithms**

# Mathematical Algorithms

## Plan Synthesis

### Types of Planning Problems

#### **Spectrum Demand Study**

**application:** introduction of a  
new service

**target:** estimate minimum  
amount of spectrum  
needed to satisfy all  
requirements

**method:** graph coloring

# Mathematical Algorithms

## Plan Synthesis

### Types of Planning Problems

#### Spectrum Demand Study

**application:** introduction of a new service

**target:** estimate minimum amount of spectrum needed to satisfy all requirements

**method:** graph coloring

#### Constrained Frequency Assignment Problems

**application:** planning of a service in a given spectrum range with individual access to spectrum

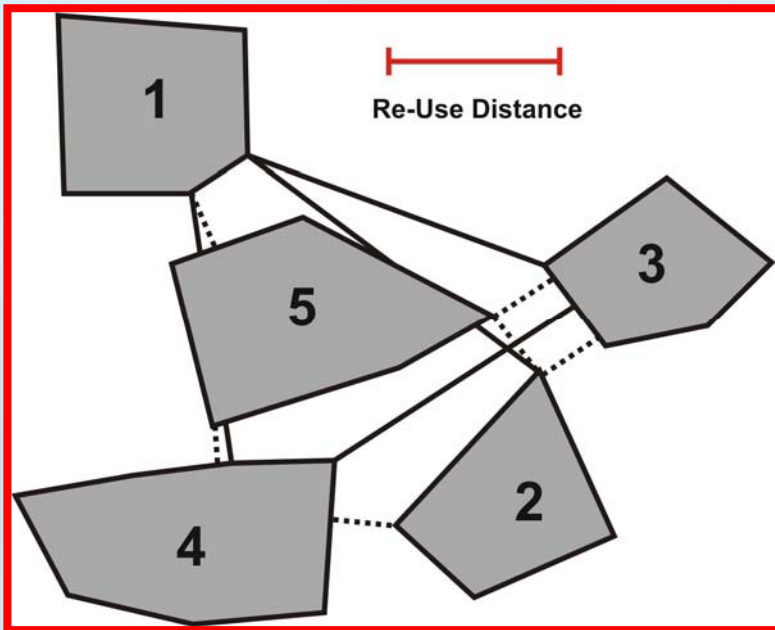
**target:** use available spectrum to fit in requirements such that interference is minimized

**method:** stochastic optimization

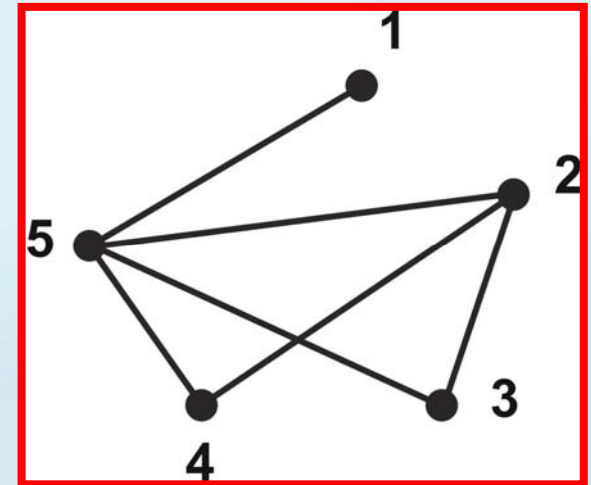
# Mathematical Algorithms

## Spectrum Demand Study

### Graph Coloring Algorithms



mapping  
to graph





# Mathematical Algorithms

## Spectrum Demand Study

### Graph Coloring Algorithms

**target:** assign a color to each vertex such that vertices connected get different colors whilst minimizing the total number of required colors

**complexity:** high dimensional combinatorial optimization problem  
N allotments, M frequencies,  
e.g. N=20, M=10 results in  $10^{20}$  different possibilities !!!

brute force approach:

0.000001 s to check one  $\rightarrow 10^{14}$  s =  $3.17 \cdot 10^6$  ys !!!



how to tackle that (NP-hard) problem ???

# Mathematical Algorithms

## Spectrum Demand Study

### Graph Coloring Algorithms

**solution:** look for almost optimal solution

**methodology:** sequential deterministic graph coloring

- generate an ordering of vertices
  - use vertex degrees: number of edges ending in a vertex
- assign frequencies in compliance with adjacency relations

# Mathematical Algorithms

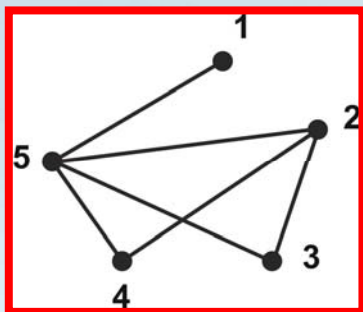
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vertex	5	2	4	3	1
degree	4	3	2	2	1
color					

# Mathematical Algorithms

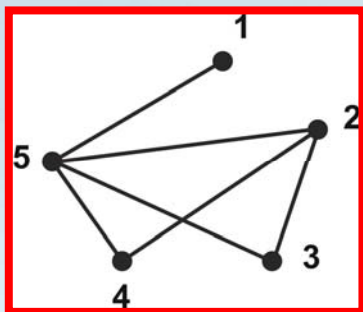
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vertex	5	2	4	3	1
degree	4	3	2	2	1
color	0	1	2	2	1

→ at RRC-06 many different orderings have been employed

# Mathematical Algorithms

## Constrained Frequency Assignment Problems

spectrum demand study limited help for constrained problems:

- spectrum accessibility globally restricted and individualized

# Mathematical Algorithms

## Constrained Frequency Assignment Problems

spectrum demand study limited help for constrained problems:

- spectrum accessibility globally restricted and individualized
- individual constraints overriding the results of the compatibility analysis have to be taken into account (administrative declarations)

# Mathematical Algorithms

## Constrained Frequency Assignment Problems

spectrum demand study limited help for constrained problems:

- spectrum accessibility globally restricted and individualized
- individual constraints overriding the results of the compatibility analysis have to be taken into account (administrative declarations)
- graph coloring algorithms adapted to particular boundary conditions and constraints

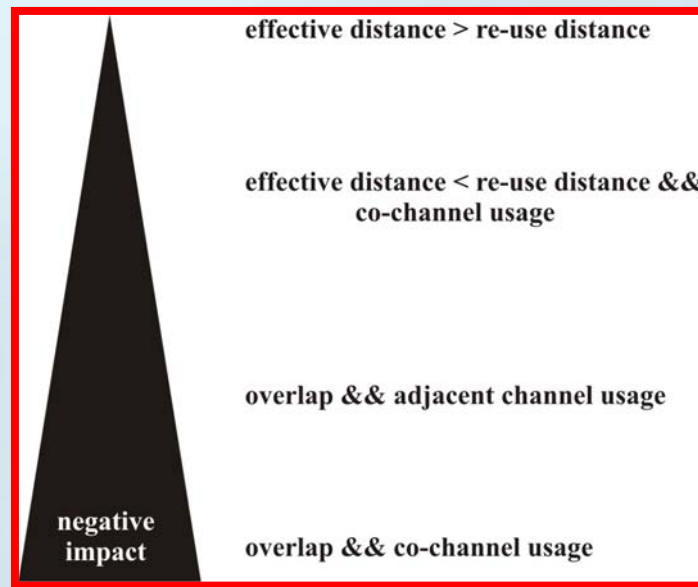
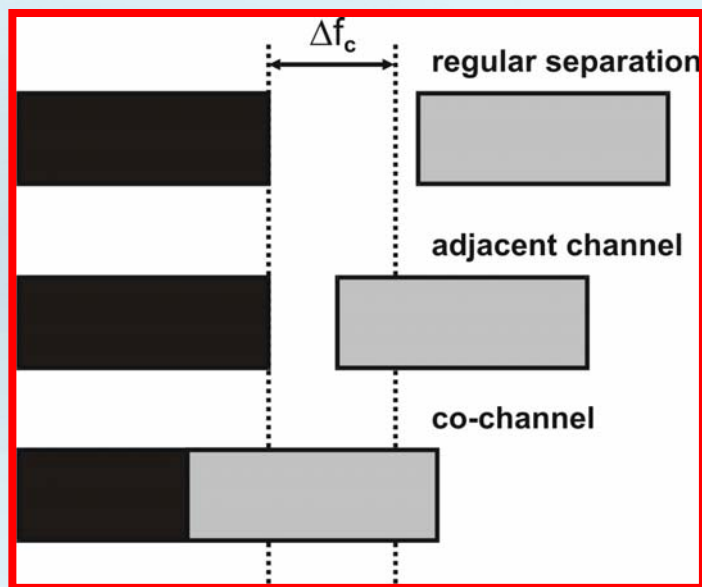
# Mathematical Algorithms

## Constrained Frequency Assignment Problems

### Impact in terms of Interference

several layers of allotment mean overlapping allotments

→ co-channel usage vs. adjacent channel usage





# Mathematical Algorithms

## Constrained Frequency Assignment Problems

spectrum demand study limited help for constrained problems:

- spectrum accessibility globally restricted and individualized
- individual constraints overriding the results of the compatibility analysis have to be taken into account (administrative declarations)
- graph coloring algorithms adapted to particular boundary conditions and constraints



stochastic optimization strategies can help  
example : Great Deluge Algorithm

# Mathematical Algorithms

## Constrained Frequency Assignment Problems

### Stochastic Optimization

stochastic optimization can be applied if

- state or configuration of a system can be fully described by a set of parameters  
(here: configuration corresponds to assigning a frequency to all requirements)
- a quality function can be defined to assess a configuration of the system  
(here: number of used channels, conflicting frequency assignments )
- „neighborhood“ can be defined in configuration space  
(here: changing a single frequency results in a similar quality)
- iterative stochastic search in configuration space can be defined  
(here: start with one frequency assignment, generate a new one out of previous)
- dynamically changing access to states in configuration space can be defined

# Mathematical Algorithms

## Constrained Frequency Assignment

### Great Deluge Algorithm



# Mathematical Algorithms

## Constrained Frequency Assignment

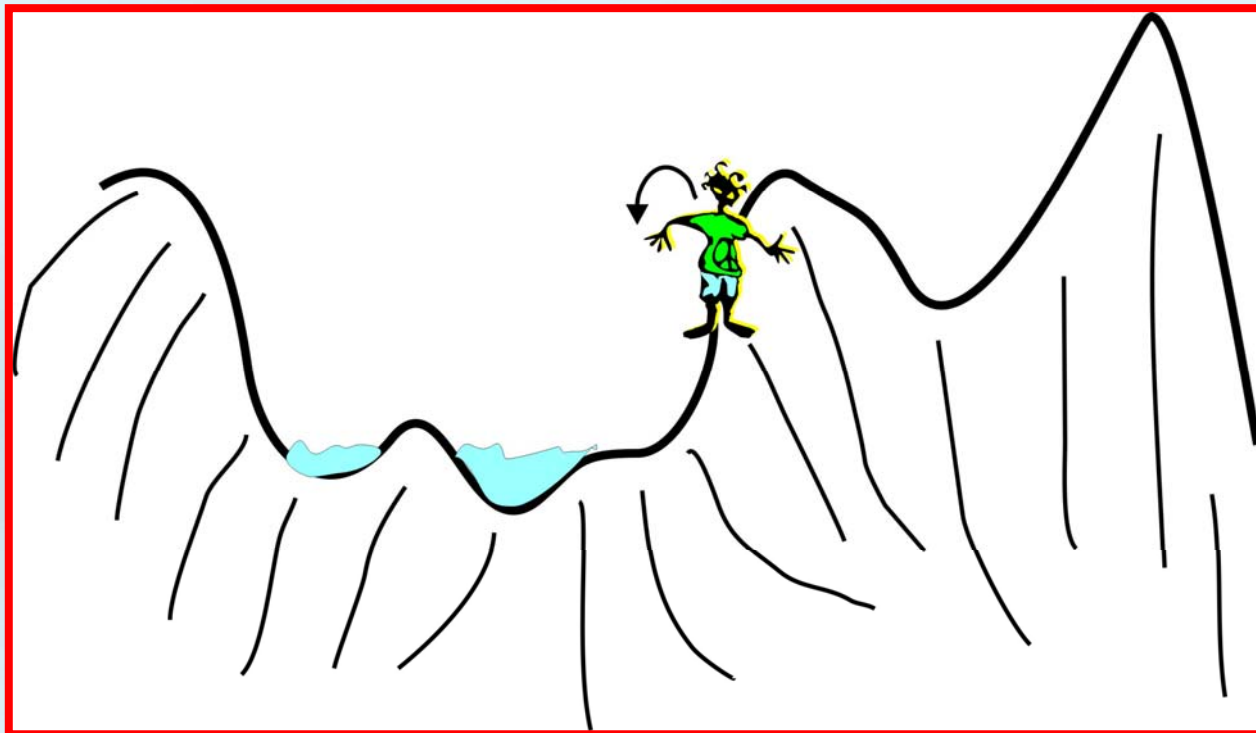
### Great Deluge Algorithm



# Mathematical Algorithms

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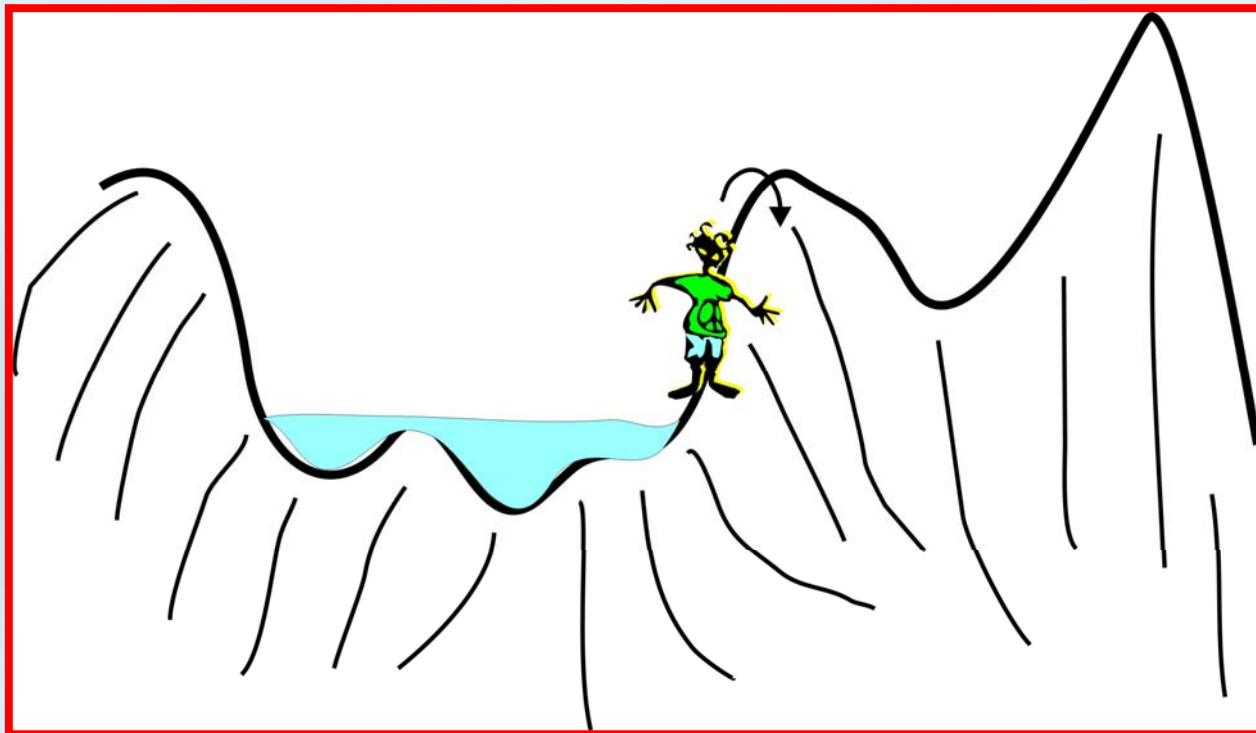
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# Mathematical Algorithms

## Constrained Frequency Assignment

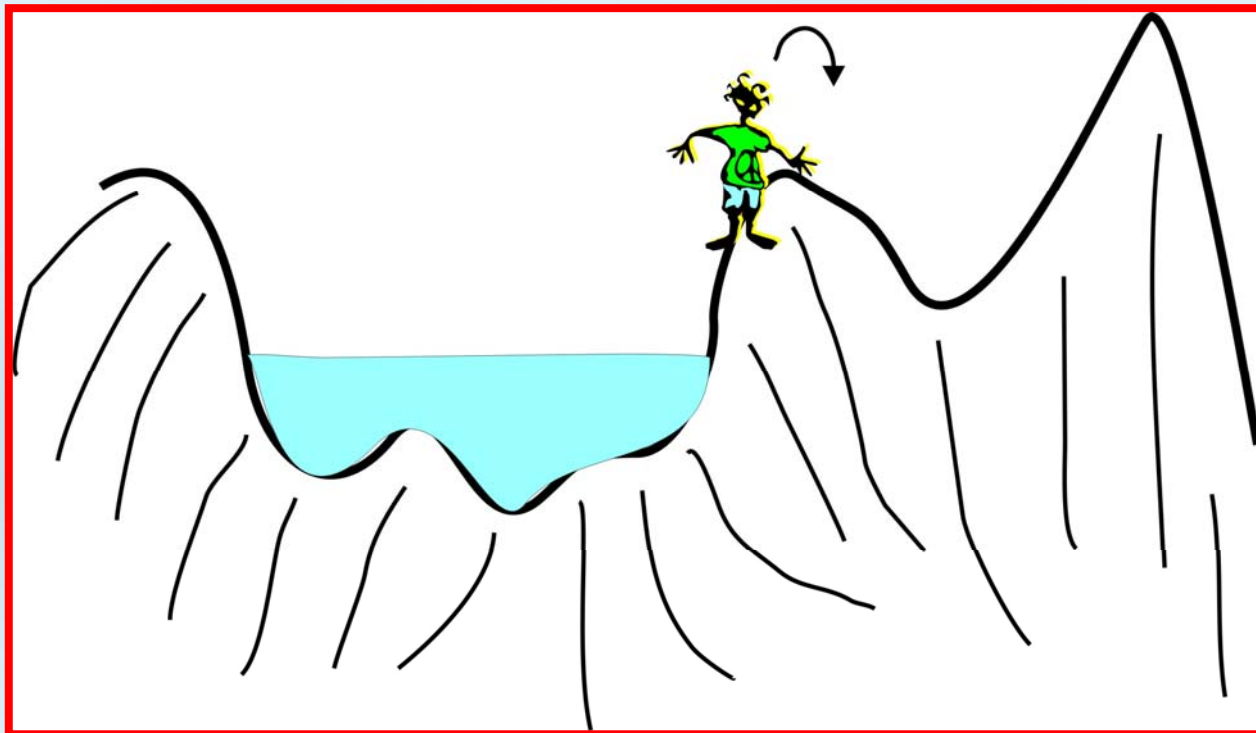
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# Mathematical Algorithms

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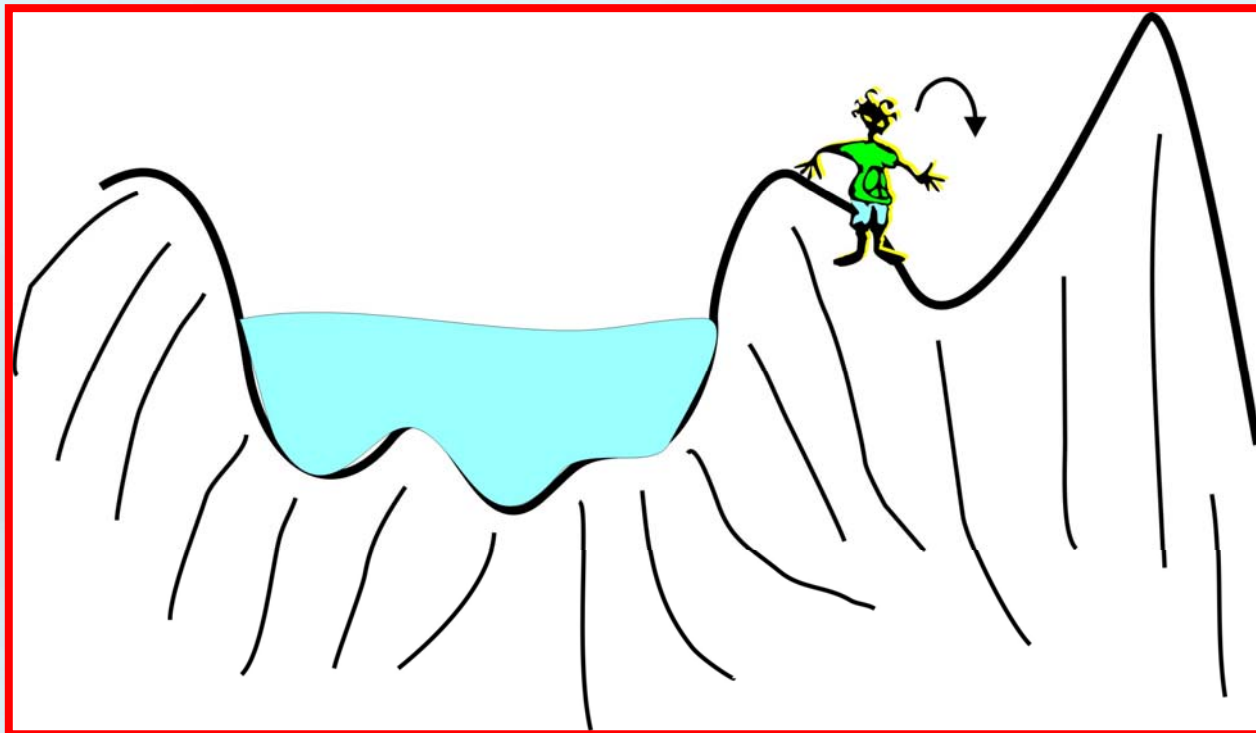
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# Mathematical Algorithms

## Constrained Frequency Assignment

### Great Deluge Algorithm

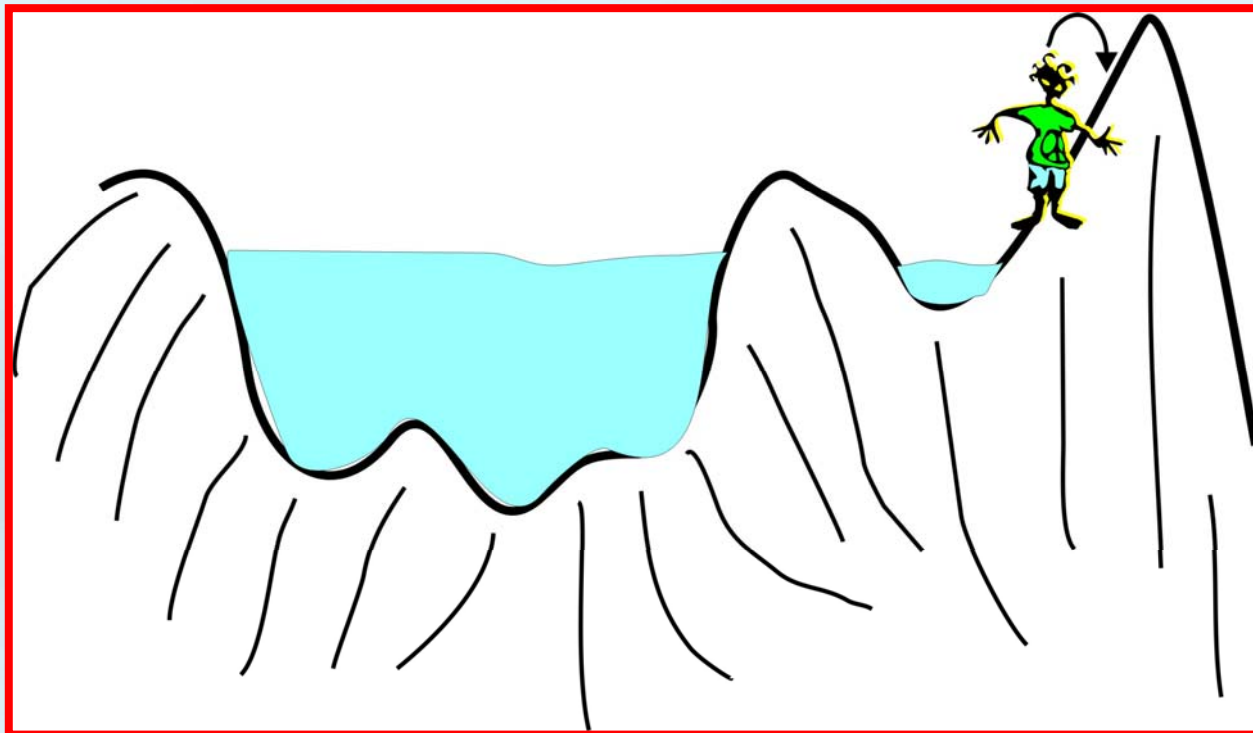




# Mathematical Algorithms

## Constrained Frequency Assignment

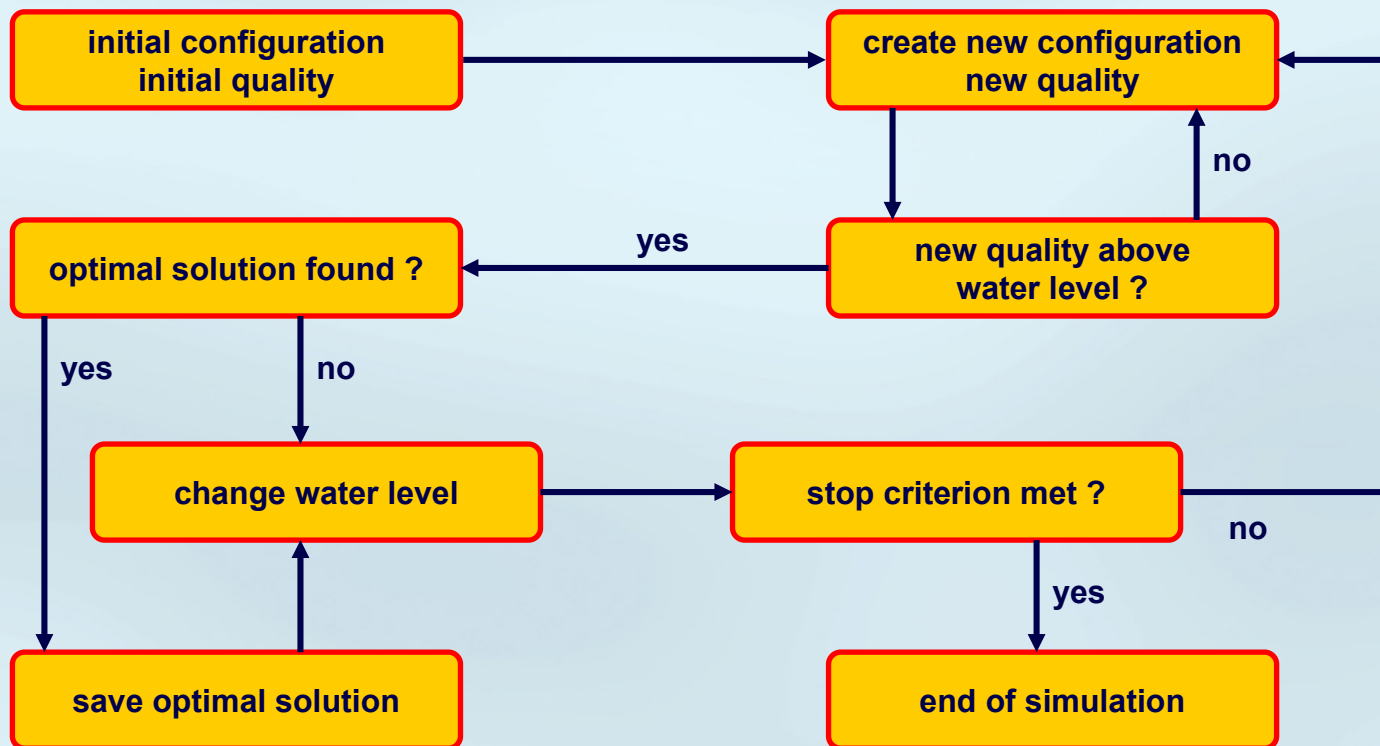
### Great Deluge Algorithm



# Mathematical Algorithms

## Constrained Frequency Assignment

### Great Deluge Algorithm



# Mathematical Algorithms

## Constrained Frequency Assignment

### Quality Function

assessment of mutual impact of two co-channelled allotments:

$$\Delta_{ij} = RU - ED(i, j)$$

measure of overlap between allotment areas:

$$\psi_{ij} = \frac{2 \cdot \text{size of overlapping area}}{\text{area}(i) + \text{area}(j)}$$

# Mathematical Algorithms

## Constrained Frequency Assignment

### Quality Function

$$\Delta_{ij} = \begin{cases} 0 & , \quad RU < ED(i,j) \\ RU - ED(i, j) & , \quad RU \geq ED(i,j) \geq 0 \\ RU + 100 \cdot \psi_{ij} & , \quad \text{for overlapping areas} \end{cases}$$


$$\Delta = \sum_{i,j} \Delta_{ij}$$

summation of pair contributions

→ example: planning exercise submitted to ECC-TG4

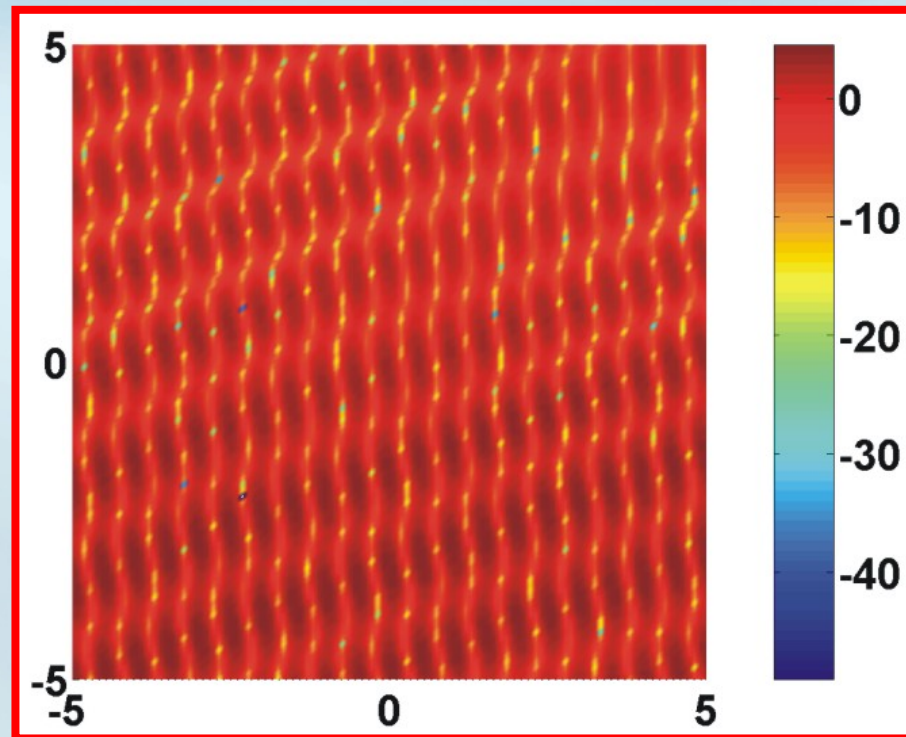
## Concluding Remarks

- **pair contributions:** „solution globally optimal but locally a disaster“  
→ cluster based quality function ?
- **reduction of computational times by means of parallel computing**
- **stochastic optimization also applicable to network planning:**
  - site selection
  - optimization of erp, antenna pattern, time delay
  - area or population coverage
  - network cost optimization

**Thank you very much  
for your attention !**

# Planning Parameters

## Coverage Prediction

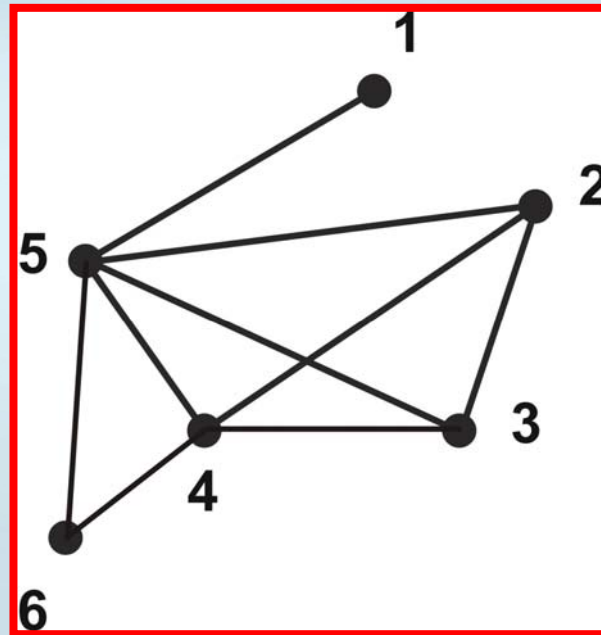


**multipath environment gives rise to spatial variation of field strength**

# Mathematical Algorithms

## Spectrum Demand Study

### Clique Analysis



maximum clique: { 2, 3, 4, 5 }

→ vertices are members of several cliques!



# Mathematical Algorithms

## Spectrum Demand Study

### Clique Analysis

quality of solution: is that a good or bad result ?

idea: what is the chromatic number  $\chi$  of the graph?  
→ investigate the cliques of the graph

clique: other word for complete graph referring to a graph  
where all vertices have the same degree  $\kappa$

estimate: the vertex degree of the maximum clique  $\kappa_{\max}$  is linked to the  
chromatic number  $\chi$  :

$$\chi \geq \kappa_{\max} + 1$$

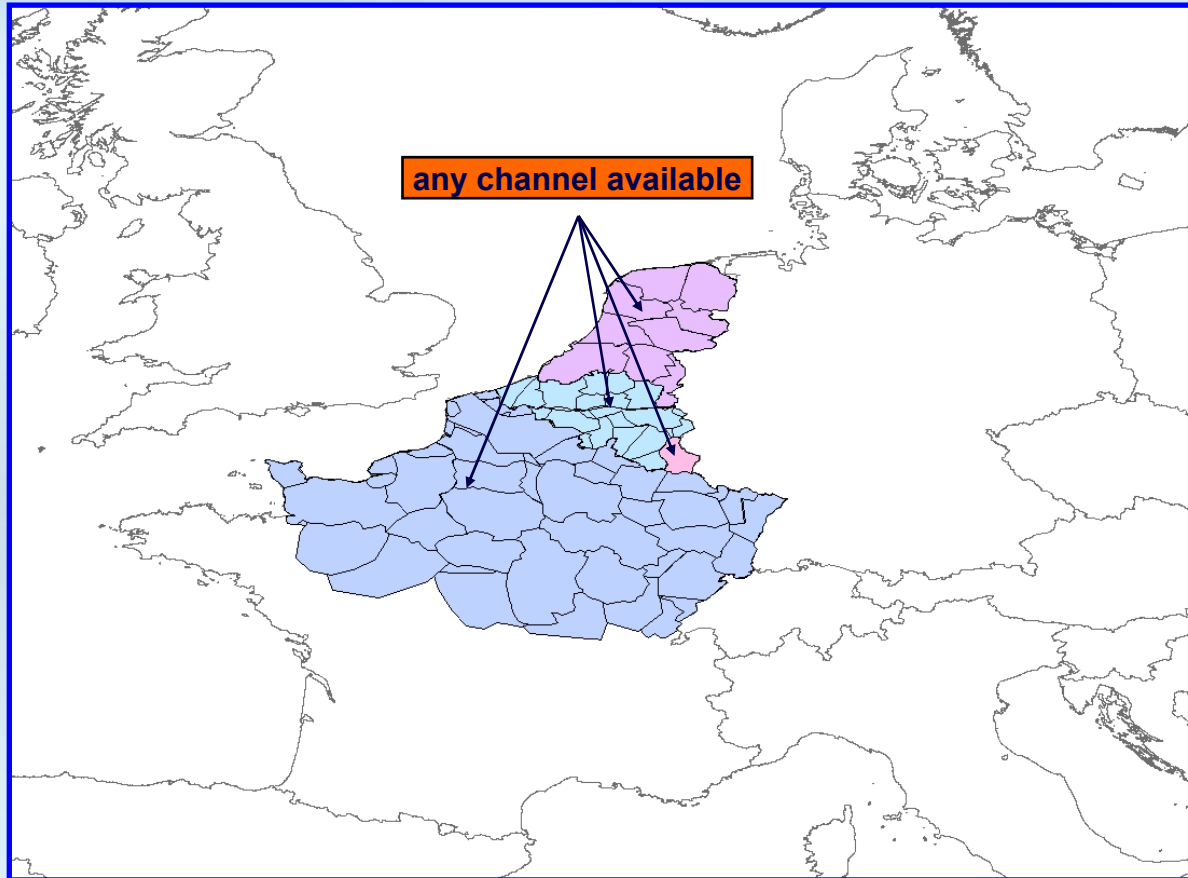


finding the maximum clique is NP-hard, too.  
deterministic algorithm works for modest number of vertices

# Mathematical Algorithms

## Spectrum Demand Study

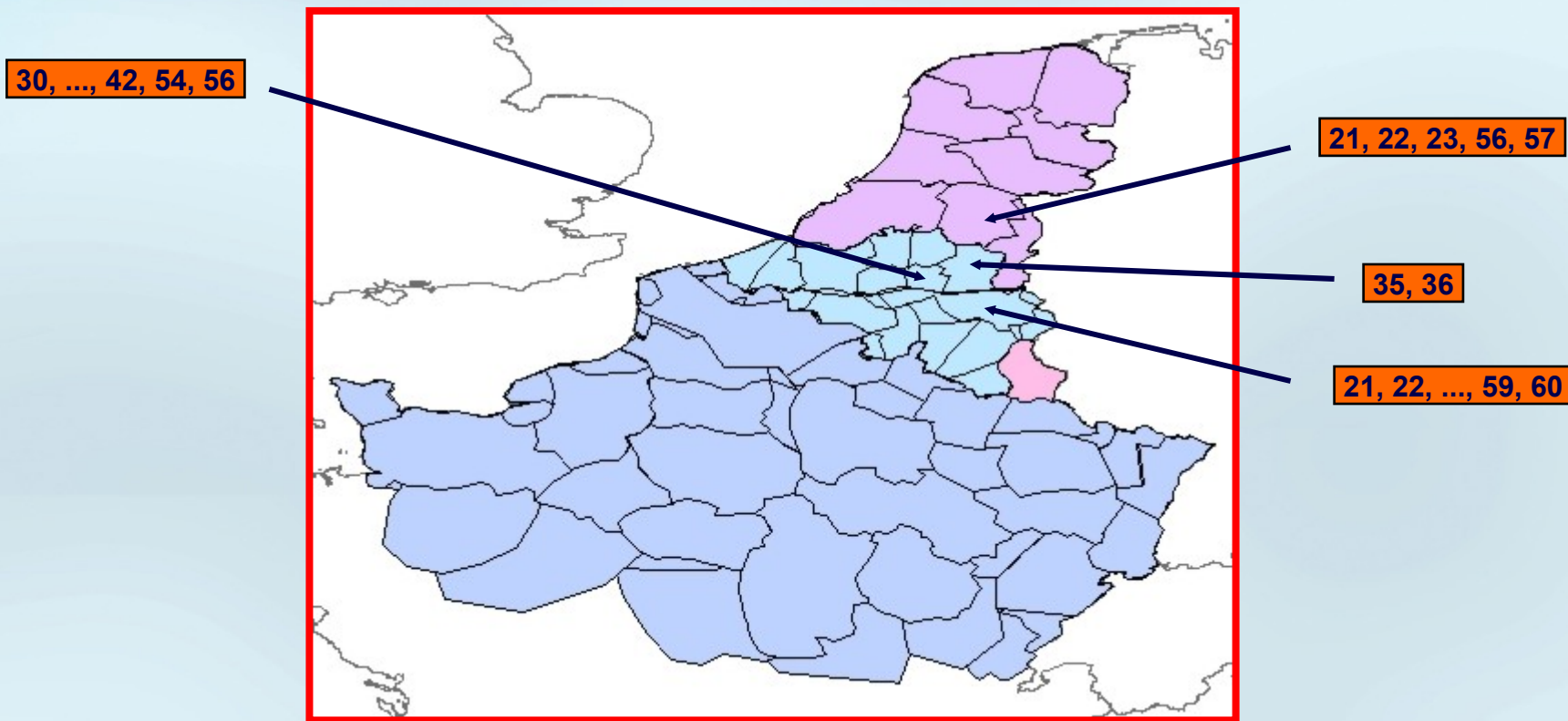
no boundary conditions



# Mathematical Algorithms

## Constrained Frequency Assignment

### Restricted Access to Spectrum

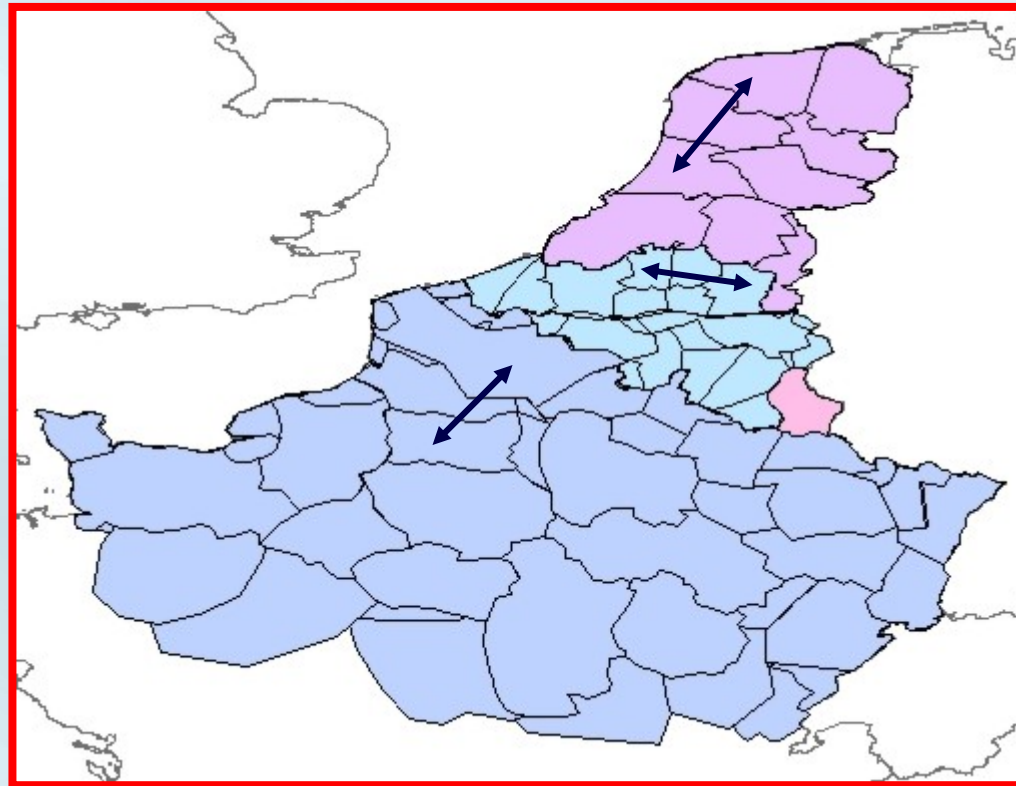


# Mathematical Algorithms

## Constrained Frequency Assignment

### Administrative Declarations

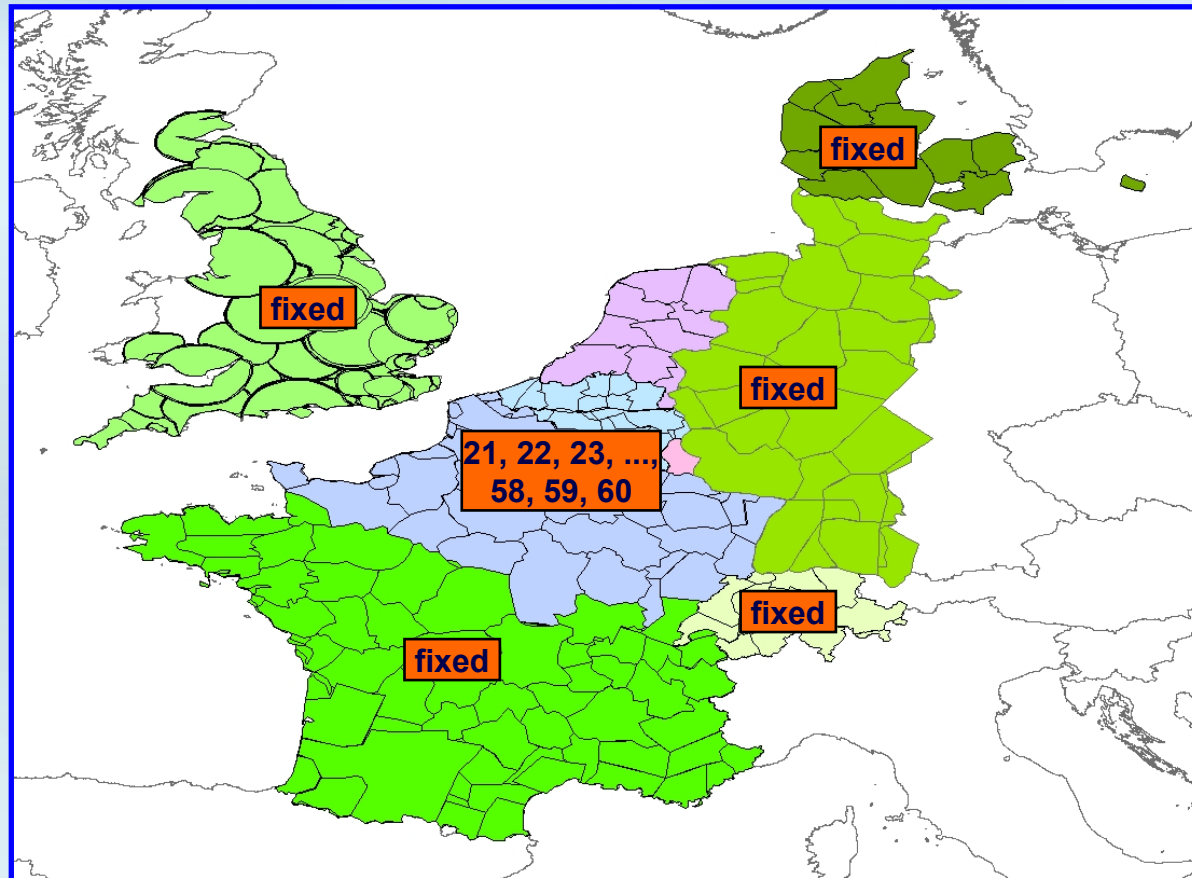
allotments are allowed to share a channel in contrast to result of compatibility analysis



# Mathematical Algorithms

## Constrained Frequency Assignment

### Case 1 : Fixed Boundary Conditions



# Mathematical Algorithms

## Constrained Frequency Assignment

### Case 2 : Restricted Access to Spectrum

